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B. Tech. 3rd Semester (CSE) Examination – March, 2021

MATHEMATICS - III (Multivariable Calculus and Differential Equations)

Paper : BSC-MATH-203-G

Time : Three Hours] *[Maximum Marks : 75*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, selecting ***one*** question from each Unit. Question No. 1 is ***compulsory***. All questions carry equal marks.

1. (a) Evaluate $\lim\limits_{\substack{x\to 0\\ y\to 0}} \frac{xy}{x^2+y^2}$. 15

(b) If $u = x^2 + y^2 + z^2$ and $x = e^{2t}$, $y = e^{2t}\cos 3t$ and $z = e^{2t}\sin 3t$. Find $\frac{du}{dt}$.

(c) Evaluate $\iint r\sin\theta\, dr\, d\theta$ over the cardioid $r = a(1-\cos\theta)$ above the initial line.

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(d) Solve $x\log x\frac{dy}{dx} + y = \log x^2$.

(e) Solve $\frac{d^4y}{dx^4} + a^4y = 0$

(f) Solve $\frac{d^2y}{dx^2} + y = \tan x$ by the method of variation of parameter.

UNIT – I

2. (a) If $x^x y^y z^z = C$, show that at $x = y = z$, $\frac{\partial^2 z}{\partial x\,\partial y} = -(x\log ex)^{-1}$ 15

(b) If $u = \tan^{-1}\frac{x^3+y^3}{x-y}$, prove that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2\cos 3u\sin u$$

3. (a) Find the minimum value of $x^2 + y^2 + z^2$, given that $xyz = a^3$. 15

(b) If $u = a^3x^2 + b^3y^2 + c^3z^2$, where $x^{-1} + y^{-1} + z^{-1} = 1$, find the stationary value of u.

UNIT – II

4. (a) Evaluate $\int_0^1\int_0^{\sqrt{1-y^2}} x^3y\, dx\, dy$ by changing the order of integration. 15

(b) Evaluate $\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)}\,dx\,dy$ by using the transformation $x = r\cos\theta$ and $y = r\sin\theta$.

5. (a) Find by double integration, the area lying inside the circle $r = a\sin\theta$ and outside the cardioid $r = a(1 - \cos\theta)$. 15

(b) Evaluate $\int_0^a\int_0^x\int_0^{x+y} e^{x+y+z}\,dz\,dy\,dx$

UNIT – III

6. (a) Solve $\frac{dy}{dx} - \frac{\tan y}{1+x} = (1+x)e^x \sec y$. 15

(b) Solve $(x^2y^2 + xy + 1)y\,dx + (x^2y^2 - xy + 1)x\,dy = 0$

7. (a) A steam pipe 20 cm in diameter contains steam at 150° C and is covered with asbestos 5 cm thick. The outside temp is kept at 60°C. By how much the thickness of the covering be increased in order that the rate of heat loss should be decreased by 25% ? 15

(b) Find the orthogonal trajectories of the family of confocal and coaxial parabolas $r = 2a/(1 + \cos\theta)$.

UNIT – IV

8. (a) Solve $(D^3 - D)y = 2x + 1 + 4\cos x + 2e^x$ 15

(b) Solve $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^2 + 2\log x$

9. (a) Solve the following simultaneous equation : 15

$\frac{dx}{dt} + 2x - 3y = 5t$; $\frac{dy}{dt} - 3x + 2y = 2e^{2t}$

(b) An e.m.f. $E\sin pt$ is applied at $t = 0$ to a circuit containing a capacitance C and inductance L. The current i satisfies the equation

$L\frac{di}{dt} + \frac{1}{C}\int i\,dt = E\sin pt$. If $p^2 = 1/LC$ and initially the current i and the charge q are zero, show that the current at time t is $\left(Et/2L\right)\sin pt$, where $i = \frac{dq}{dt}$.